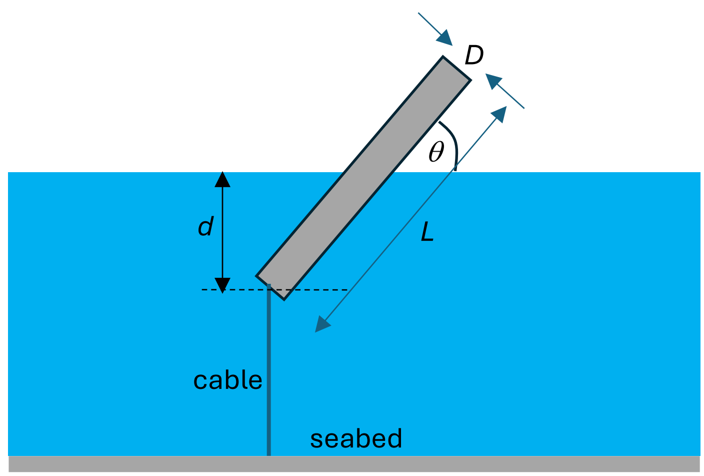
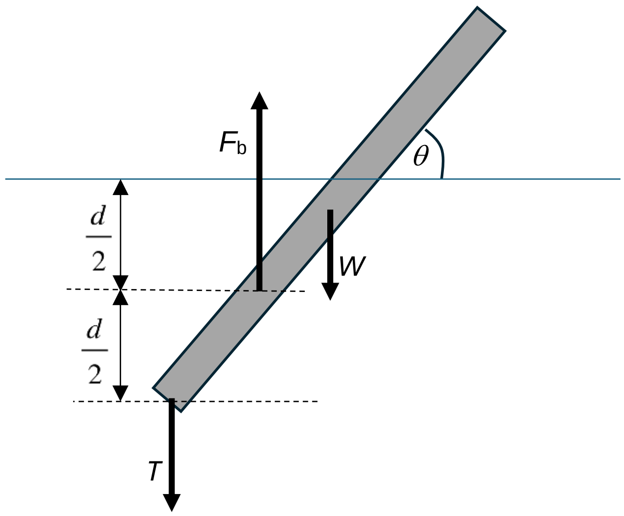
Teacher notes
Topic A

A problem on torques leading to an IA

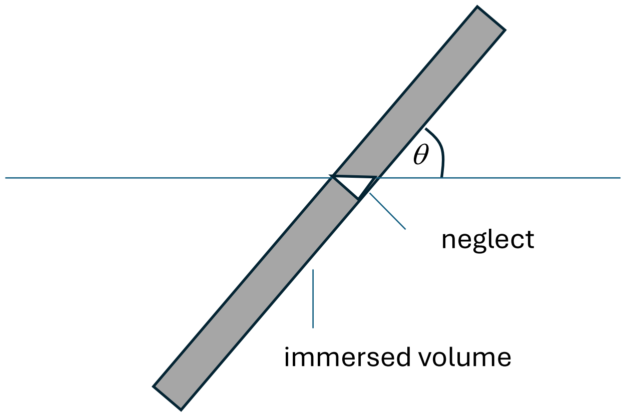
A uniform cylindrical buoy of density ρ_b is tethered to the seabed with a cable. The buoy has length *L*, diameter *D* and makes an angle θ with the surface of the water. The density of water is ρ_w. The bottom of the buoy is at depth *d* below the surface of the water.



The force diagram shows the three forces acting on the buoy and the point of application of each force: the weight *W*, the tension *T* and the buoyancy force F_b.



The weight of the buoy is given by $W=\rho_b Vg$. The volume is $V=\text{area}\times\text{height}=\frac{\pi D^2}{4}\times L$ and therefore $W=\rho_b g\frac{\pi D^2}{4}L$. The buoyancy force F_b is given by $F_b=\rho_w V_{imm}g$. To calculate the immersed volume ignore the volume colored white in the figure below.



Then $V_{imm}=\frac{\pi D^2}{4}\times\frac{d}{\sin\theta}$. Therefore $F_b=\rho_w g\frac{\pi D^2}{4}\frac{d}{\sin\theta}$.

Taking torques about an axis through the point of application of the tension force T we find:

$$\rho_b g\frac{\pi D^2}{4}L\times\frac{L}{2}\cos\theta=\rho_w g\frac{\pi D^2}{4}\frac{d}{\sin\theta}\times\frac{d}{2\tan\theta}$$

i.e.

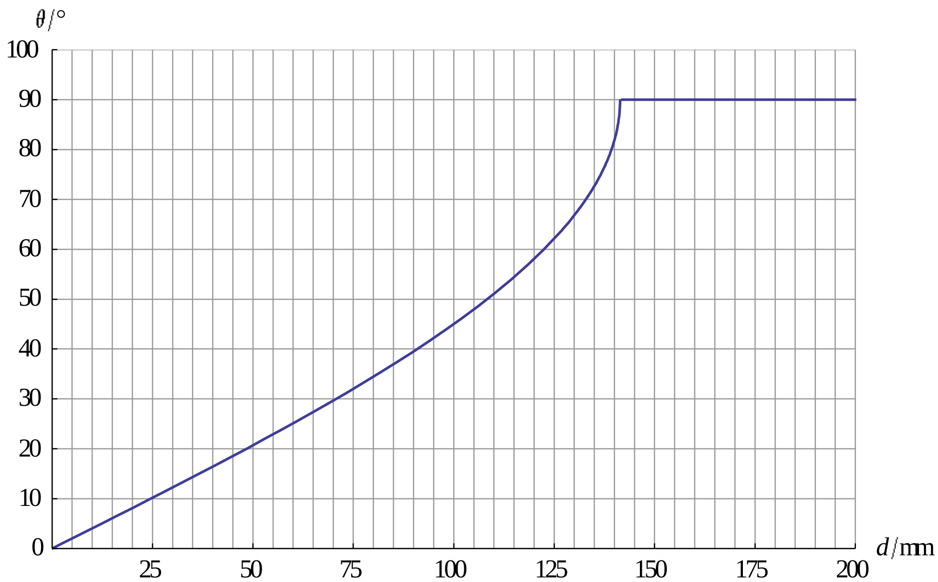
$$\rho_b L^2=\rho_w\frac{d^2}{\sin^2\theta}$$

Finally,

$$\sin\theta=\frac{d}{L}\sqrt{\frac{\rho_w}{\rho_b}}$$

It is interesting that the angle does not depend on the diameter D of the buoy.

This can lead to an interesting IA. You can investigate the dependence of the angle on d or L or ρ_b. For example, for fixed L and ρ_b a graph of $\sin\theta$ against d would be a straight line. A graph of θ against d would be (this is for L = 200 mm and $\frac{\rho_w}{\rho_b}=2$):



You need to pay attention to

- a method by which the angle θ may be accurately measured.
- why angles near 90° are more difficult to measure.
- if you are investigating the dependence of the angle on *d* how would you deduce the value of ρ_0 and the value of *L*?
- give a physical reason why $\theta \to 90^\circ$ as the depth increases.